# Refinement of the crystal structure of $\mathrm{NiCl}_{\mathbf{2}}$ and of unit-cell parameters of some anhydrous chlorides of divalent metals. By Adolfo Ferrari, Antonio Braibanti and Gino Bigliardi, 1stituto di Chimica generale and Istituto di Mineralogia, Universita di Parma, Italy 

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After the chemical researches of Sandonnini (1914) on solid solutions between LiCl and $\mathrm{MgCl}_{2}$, Bruni \& Ferrari (1925, 1926, 1927), Ferrari (1927), Ferrari, Celeri \& Giorgi (1929) studied the same problem on a structural basis and found that complete solubility is connected with the presence in $\mathrm{MgCl}_{2}$ of a multiple rhombohedral cell with $\alpha=90^{\circ}$ (i.e. the cell is a cube) of the same size as the doubled cubic cell of LiCl . In the cube half of the lithium positions are occupied by magnesium atoms, leaving chlorine atoms unaffected. They found also that $\mathrm{MnCl}, \mathrm{FeCl}_{2}, \mathrm{CoCl}_{2}, \mathrm{NiCl}_{2}$ and $\mathrm{CdCl}_{2}$ have the same structure as $\mathrm{MgCl}_{2}$. Pauling (1929), and Pauling \& Hoard (1930), were able to find the rhombohedral unit cell and space group of $\mathrm{CdCl}_{2}$ and to determine more precisely the positions occupied by the atoms in the unit cell. The structure was defined as C19 in Strukturbericht (vol. I, p. 742).

We have measured new photographs of the same compounds and applied new methods of extrapolation to the data obtained. Moreover, the general coordinates of the chlorine atoms in $\mathrm{NiCl}_{2}$ have been refined by the use of intensity data obtained from a single crystal.

Compounds were prepared by dehydrating the hexahydrated chlorides in gaseous HCl at high temperature ( $500-800{ }^{\circ} \mathrm{C}$ ). Magnesium chloride was obtained by heating $\mathrm{NH}_{4} \mathrm{Cl} . \mathrm{MgCl}_{2} .6 \mathrm{H}_{2} \mathrm{O}$ in flowing gaseous HCl .

The powder photographs were obtained in a 57.3 mm diameter camera; $\mathrm{Cu} K \alpha$ radiation was used $(\bar{\lambda}=$ $1.5418 \AA$ ).

## Refinement of unit-cell constants

Lattice constants were determined by the Myers \& Davies (1961) graphical method and the least-squares refinement was applied to the data obtained by the graphical method.

The least-squares normal equations were derived as follows. The usual expression for the hexagonal system:

$$
\begin{equation*}
\frac{4 \sin ^{2} \theta}{\lambda^{2}}=\frac{4}{3} \frac{\left(h^{2}+h k\right.}{a^{2}}+\frac{\left.k^{2}\right)}{}+\frac{l^{2}}{c^{2}} \tag{1}
\end{equation*}
$$

was rewritten in the form

$$
\begin{equation*}
\frac{4 \sin ^{2} \theta}{\lambda^{2} l^{2}}=\frac{1}{a^{2}}\left[\frac{a^{2}}{c^{2}}+\frac{4}{3} \frac{\left(h^{2}+h k+k^{2}\right)}{l^{2}}\right] \tag{2}
\end{equation*}
$$

Hence

$$
\begin{equation*}
\frac{4 \sin ^{2} \theta_{c}}{\lambda^{2} l^{2}}=\frac{r^{2}+K}{a_{0}^{2}+B x} \tag{3}
\end{equation*}
$$

when one puts

$$
\frac{a_{0}}{c}=r, \frac{4}{3} \frac{\left(h^{2}+h k+k^{2}\right)}{l^{2}}=K, \quad a^{2}=a_{0}^{2}+B x
$$

with

$$
x=\frac{1}{2}\left(\frac{\cos ^{2} \theta}{\sin \theta}+\frac{\cos ^{2} \theta}{\theta}\right),
$$

the Nelson \& Riley (1945) and Taylor \& Sinclair (1945) extrapolation function.

These authors extrapolated to $x=0$ the function $a=a_{0}+B x$, but Myers \& Davies (1961) found satisfactory the extrapolation of the function $a^{2}=a_{0}^{2}+B x$ as well, because it is applied to a narrow range of $a$.
$\operatorname{Sin} \theta_{c}$ in (3) indicates the value resulting from the introduction in (2) of the values of $a_{0}^{2} / c^{2}, a_{0}^{2}$ and $B$ obtained by graphical extrapolation according to Myers \& Davies (1961), or of further values of $a_{0}^{2} / c^{2}, a_{0}^{2}$ and $B$ obtained in successive cycles of the least-squares procedure.

If one indicates $C_{c}=4 \sin ^{2} \theta_{c} / \lambda^{2} l^{2}$ and $C_{o}=4 \sin ^{2} \theta_{o} / \lambda^{2} l^{2}$ ( $\theta_{c}$ is defined as above and $\theta_{o}$ is the Bragg angle observed), the following least-squares normal equations can be obtained (partially diagonalized matrix):

$$
\begin{align*}
\Sigma\left(C_{o}-C_{c}\right) \frac{\partial C_{c}}{\partial\left(r^{2}\right)}= & \Sigma\left[\frac{1}{a_{0}^{2}+B x}\right]^{2} \Delta\left(r^{2}\right) \\
& -\Sigma\left[\frac{1}{a_{0}^{2}+B x} \frac{r^{2}+K}{\left.Q_{0}^{2}+B x\right)^{2}}\right] \Delta\left(a_{0}^{2}\right) \\
\Sigma\left(C_{o}-C_{c}\right) \frac{\partial C_{c}}{\partial\left(a_{0}^{2}\right)}= & -\Sigma\left[\frac{1}{a_{0}^{2}+B x} \frac{r^{2}+K}{\left(a_{0}^{2}+B x\right)^{2}}\right] \Delta\left(r^{2}\right)  \tag{4}\\
& +\Sigma\left[\frac{r^{2}+K}{\left(Q_{0}^{2}+B x\right)^{2}}\right]^{2} \Delta\left(Q_{0}^{2}\right) \\
\Sigma\left(C_{o}-C_{c}\right) \frac{\partial C_{c}}{\partial B}= & \Sigma\left[\frac{\left(r^{2}+K\right) x}{\left(a_{0}^{2}+B x\right)^{2}}\right]^{2} \Delta B
\end{align*}
$$

Iterative application of all the equations (4) gives the best values of $r^{2}$ and $a_{0}^{2}$ in a few cycles. The correctness of the procedure can be judged by the reasonable agreement of the final results with single-crystal data obtained for $\mathrm{NiCl}_{2}$ (Table 1).

From (4) the e.s.d.'s can also be calculated (International Tables for X-ray Crystallography, 1959).

The photograph of $\mathrm{MgCl}_{2}$ was indexed in terms of a pseudo-cubic cell, and the data then transformed to the hexagonal cell. The pseudo-cubic cell of $\mathrm{MgCl}_{2}$ has $a=10 \cdot 274 \AA$, which can be compared to the cube in LiCl having $2 a=10 \cdot 2796 \AA$ (Jevinš, Straumanis \& Karlsons, 1938).

The unit-cell constants are quoted in Table 1.

## Refinement of the structure of $\mathbf{N i C l}_{\mathbf{2}}$

The crystal used for the photographs was obtained from a solidified mass. One fragment was sealed in a Lindemann glass capillary tube. Rotation and Weissenberg photographs (Cu $K \alpha$ radiation) were taken. The rotation axis was $[10.0]$ of the hexagonal cell.

Table 1. Unit-cell constants of divalent metal chlorides $(\AA)$ Hexagonal cell, space group $R \overline{\mathbf{3}} m\left(D_{3 d}^{5}\right)$

| Compound | Ferrari et al.* |  | Grime \& Santos, $1934\left(\mathrm{CoCl}_{2}\right)$ <br> Pauling \& Hoard, $1930\left(\mathrm{CdCl}_{2}\right)$ |  | Present work |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Graphical method | Least-squares method |  |  |
|  | $a$ | $c$ |  |  | $a$ | $c$ | $a$ | $c$ |  | $a$ | $c$ |
| $\mathrm{MgCl}_{2}$ | $3 \cdot 603$ | $17 \cdot 627$ |  |  | $3 \cdot 632$ | 17.78 | $3 \cdot 632$ | $\pm 0.004$ | $17.795 \pm 0.016$ |
| $\mathrm{MnCl}_{2}$ | $3 \cdot 682$ | 17-480 |  |  | $3 \cdot 709$ | 17.59 | $3 \cdot 711$ | $\pm 0.002$ | $17.59 \pm 0.07$ |
| $\mathrm{FeCl}_{2}$ | $3 \cdot 585$ | $17 \cdot 555$ |  |  | $3 \cdot 593$ | $17 \cdot 60$ | $3 \cdot 593$ | $\pm 0.003$ | $17.58 \pm 0.09$ |
| $\mathrm{CoCl}_{2}$ | $3 \cdot 547$ | $17 \cdot 385$ | $3 \cdot 551$ | $17 \cdot 46$ | 3.553 | $17 \cdot 41$ | $3 \cdot 553$ | $\pm 0.001$ | $17.39 \pm 0.05$ |
| $\mathrm{NiCl}_{2}$ | 3.542 | 17-355 |  |  | $3 \cdot 481$ | $17 \cdot 30$ | $3 \cdot 478$ | $\pm 0.001 \dagger$ | $17.41 \pm 0.12 \dagger$ |
| $\mathrm{CdCl}_{2}$ |  |  | $3 \cdot 862$ | $17 \cdot 495$ | 3.845 | $17 \cdot 49$ | $3 \cdot 8457$ | $\pm 0.0003$ | $17.48 \pm 0.02$ |

* Bruni \& Ferrari (1925, 1926, 1927), Ferrari (1927), Ferrari, Celeri \& Giorgi (1929).
$\dagger$ From a single crystal, $a=3 \cdot 483 \pm 0 \cdot 006 \AA, c=17 \cdot 40 \pm 0 \cdot 03 \AA$.

Unit-cell data have been found to be $a=3 \cdot 483 \pm 0 \cdot 006$, $c=17.40 \pm 0.03 \AA$. Calculated and observed density were $\varrho_{c}=3.537 \mathrm{~g} . \mathrm{cm}^{-3}$ and $\varrho_{o}=3.51 \mathrm{~g} . \mathrm{cm}^{-3}$. The ( $h 0 \cdot l$ ) integrated reflexions were recorded by an integrating Weissenberg camera and measured by a microdensitometer. Polarization and Lorentz corrections were introduced. Absorption corrections were calculated by Ferrari, Braibanti \& Tiripicchio's (1961) method; a contribution, due to the Lindemann tube, deduced from a curve for cylindrical specimens was added to the transmission factors calculated. Atomic form factors were evaluated graphically from the Thomas \& Umeda (1957) data. During refinement, anomalous dispersion (International Tables for X-ray Crystallography, 1961) and secondary extinction (Pinnock, Taylor \& Lipson, 1956) were taken into account.

Ni is in position (a) and Cl in position (c) of the space group $R \overline{3} m$. Only the positional parameter $z_{\mathrm{Cl}}$ and the thermal parameters were refined by the least-squares method.

The resulting chlorine coordinate is $z_{\mathrm{Cl}}=0.2551 \pm$ 0.0008 . The variations of the thermal parameters from the isotropic value ( $B=2 \cdot 0 \pm 0 \cdot 2 \AA^{2}$ ) are within the e.s.d.'s.

The structure (Strukturbericht, vol. I, p. 742) is formed by octahedra [ $\mathrm{NiCl}_{6 / 3}$ ] grouped in layers. The metalhalogen distance is $\mathrm{Ni}-\mathrm{Cl}=2 \cdot 426 \pm 0.008 ~ \AA$, which is somewhat shorter than the sum of the ionic radii. The distances between two chlorine atoms are $\mathrm{Cl} \cdots \mathrm{Cl}=$ $3 \cdot 385 \pm 0.016 \AA$ and $3 \cdot 483 \pm 0.006 \AA$ within each octahedron and $\mathrm{Cl} \cdots \mathrm{Cl}=3 \cdot 674 \pm 0.016 \AA$ between different octahedra. The latter distance is in good agreement with twice the van der Waals radius of chlorine. The angles $\mathrm{Cl}-\mathrm{Ni}-\mathrm{Cl}$ are $89 \pm 3^{\circ}$ and $92 \pm 3^{\circ}$.

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A demonstration of a moiré pattern and a pure twist boundary model. By H. Ocken and E. N. Aqua, Hammond Metallurgical Laboratory, Yale University, New Haven, Connecticut, U.S.A.
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This communication describes interference phenomena resulting from two-dimensional point networks which can be interpreted either in terms of moiré patterns or dislocation models of pure twist boundaries.

Moiré patterns may be described as coarse arrange-
ments produced by the superposition and, in this case, rotation of identical fine networks. The coarse patterns are enlarged versions of the fine ones. Using Gevers's (1962) notation, the size of the magnified moire image, $A$, is related to the separation distance of the fine net-

